

## E:Dist Max

(Editorial: ynymxiaolongbao)

$$\begin{aligned} & |x_i - x_j| + |y_i - y_j| \\ &= \max(x_i - x_j, x_j - x_i) + \max(y_i - y_j, y_j - y_i) \\ &= \max((x_i - x_j) + (y_i - y_j), (x_i - x_j) + (y_j - y_i), (x_j - x_i) + (y_i - y_j), (x_j - x_i) + (y_j - y_i)) \\ &= \max(x_i + y_i - (x_j + y_j), x_i - y_i - (x_j - y_j), -(x_i - y_i) + (x_j - y_j), -(x_i + y_i) + x_j + y_j) \\ &= \max(|x_i + y_i - (x_j + y_j)|, |x_i - y_i - (x_j - y_j)|) \end{aligned}$$

Therefore, let  $z_i = x_i + y_i$  and  $w_i = x_i - y_i$ , then  $|x_i - x_j| + |y_i - y_j| = \max(|z_i - z_j|, |w_i - w_j|)$ . This transformation is called "45-degree rotation," which is commonly used in problems that deal with Manhattan distances. With this formula, the answer can be represented as follows:

$$\begin{aligned} & \max_{1 \leq i \leq N, 1 \leq j \leq N} |x_i - x_j| + |y_i - y_j| \\ &= \max_{1 \leq i \leq N, 1 \leq j \leq N} \max(|x_i + y_i - (x_j + y_j)|, |x_i - y_i - (x_j - y_j)|) \\ &= \max\left(\max_{1 \leq i \leq N, 1 \leq j \leq N} |z_i - z_j|, \max_{1 \leq i \leq N, 1 \leq j \leq N} |w_i - w_j|\right) \\ &= \max\left(\max_{1 \leq i \leq N} z_i - \min_{1 \leq i \leq N} z_i, \max_{1 \leq i \leq N} w_i - \min_{1 \leq i \leq N} w_i\right) \end{aligned}$$

Since the maximum and minimum values of  $z_i$  and  $w_i$  can be obtained in a total of  $O(N)$  time, one can derive the answer from them too.

Sample Code